

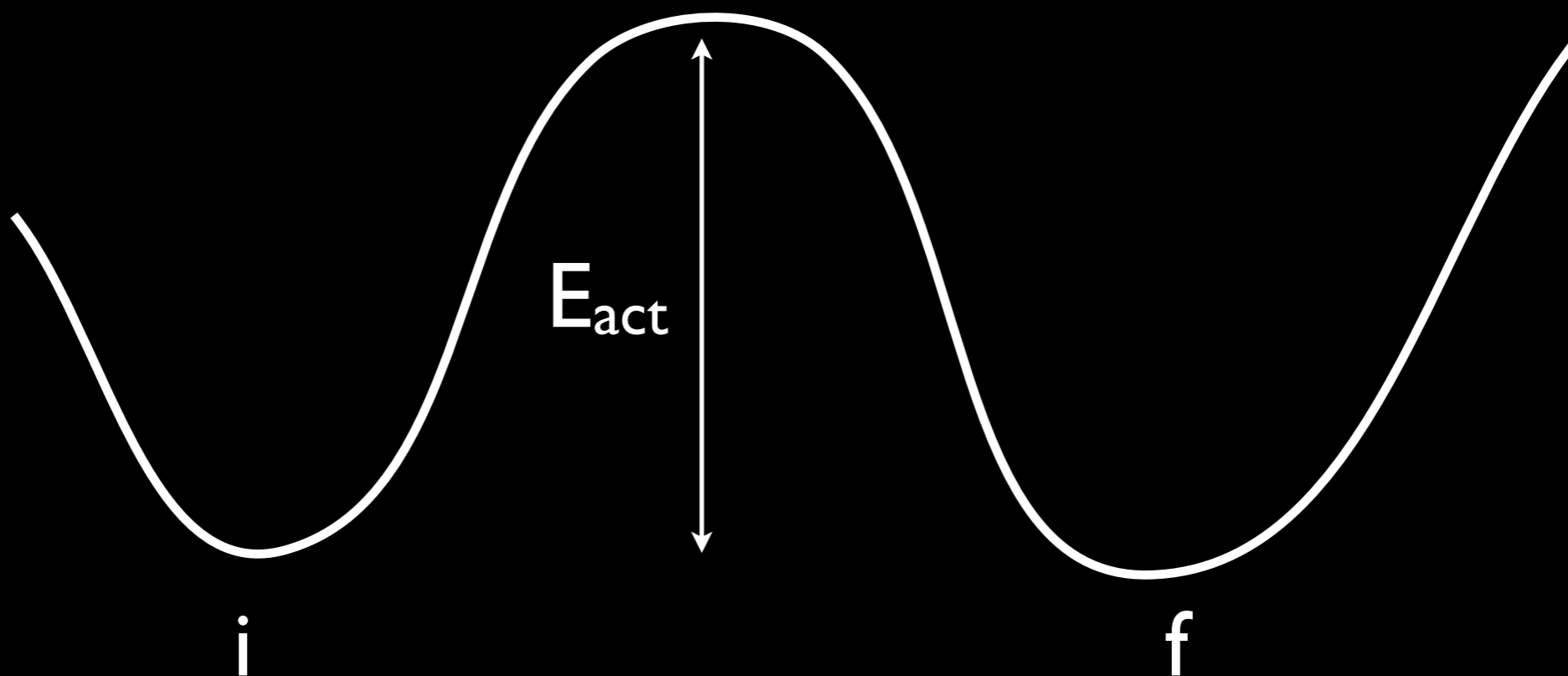
Modelling defect migration

Modelling rare events

Jump probability/reaction constant



$$R \propto e^{-\frac{G_{act}}{kT}}$$



Arrhenius law

$$R \propto e^{\frac{-G_{act}}{kT}} = e^{\frac{-(E_{act}-ST)}{kT}}$$

$$\ln(D) = \ln(D_0) - \frac{E_{act}}{kT}$$

Reaction rate or diffusion
coefficient

?

Rare events?

$$R \propto e^{-\frac{G_{act}}{kT}}$$

$$k=8.617\ 3324(78)\times 10^{-5}\text{ eVK}^{-1}$$

$$G_{act}\sim 1\text{ eV}$$

?

Rare events?

$$R \propto e^{-\frac{G_{act}}{kT}}$$

$$k=8.617\ 3324(78)\times 10^{-5}\text{ eVK}^{-1}$$

$$G_{act}\sim 1\text{ eV}$$

$$1\text{ eV} \sim 10000\text{ K!!!!!!!!!!!!}$$

atomic diffusion

Einstein law

$$D \equiv \frac{1}{2d} \lim_{t \rightarrow \infty} \frac{\langle [r(t_0 + t) - r(t_0)]^2 \rangle}{t}$$

computationally affordable for T~T fusion

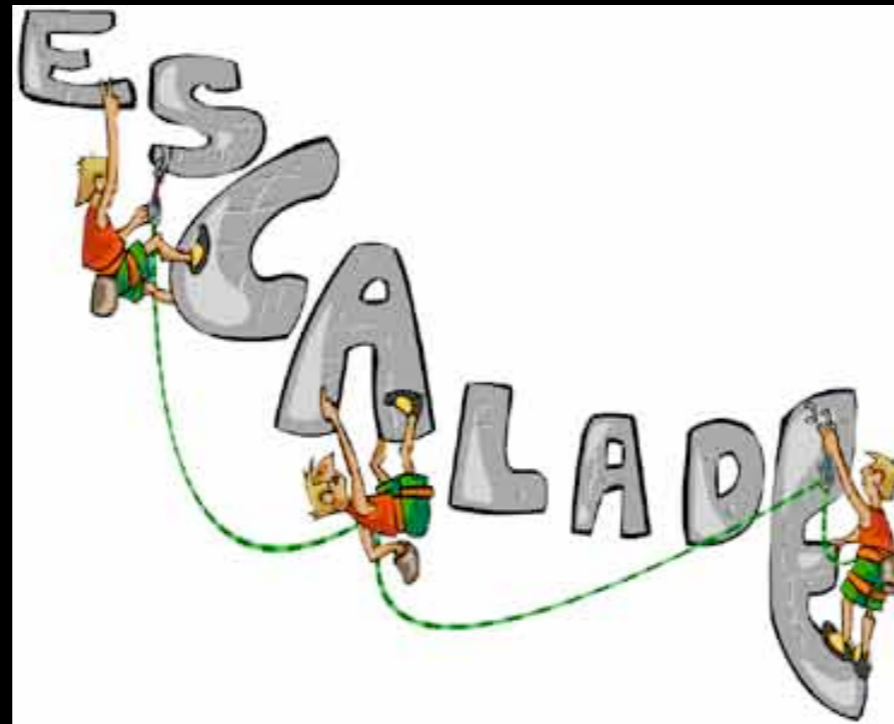
?

- Que pasaria si los atomos estan solo oscilando alrededor de la posicion de equilibrio?
- Y si los atomos se mueven como en un liquido?

Modelling?

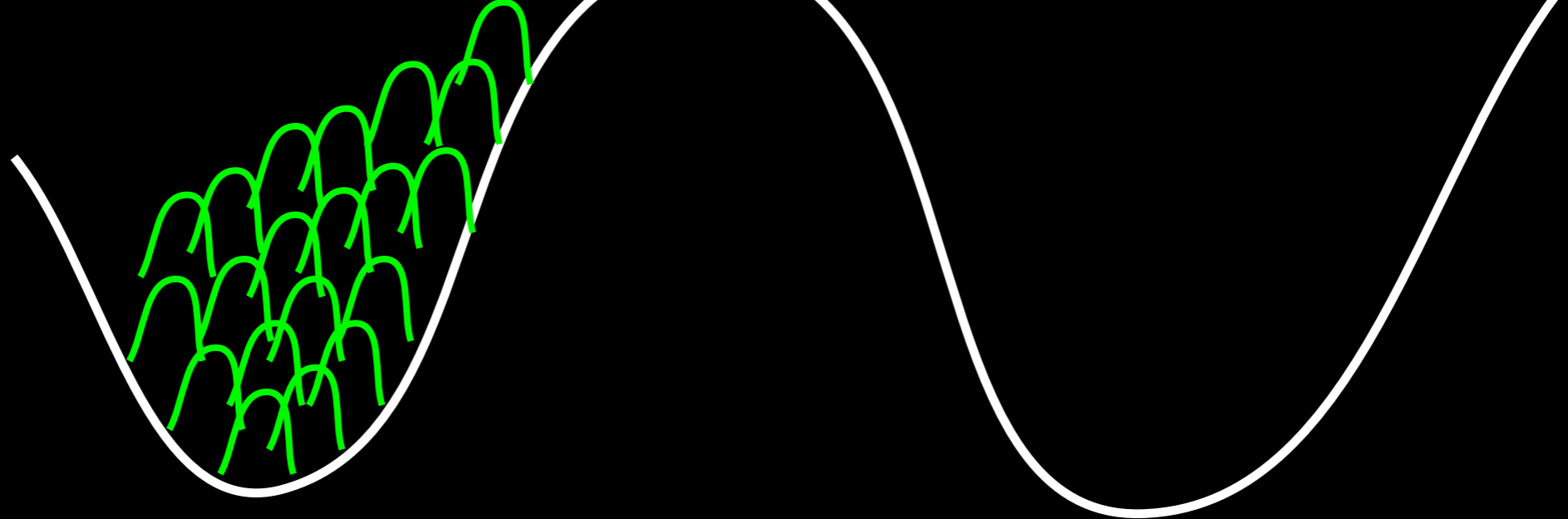
“Force the atoms to climb”

Drag like
approaches



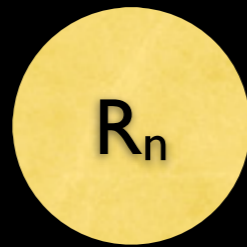
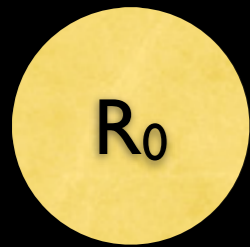
Nudged Elastic Band

“Fill the wells”



METADYNAMICS

Drag method



- 1) identification of drag coordinate
- 2) the forces parallel to the drag direction are inverted
-> forcing the system to climb
- 3) minimisation algorithm on parallel velocities

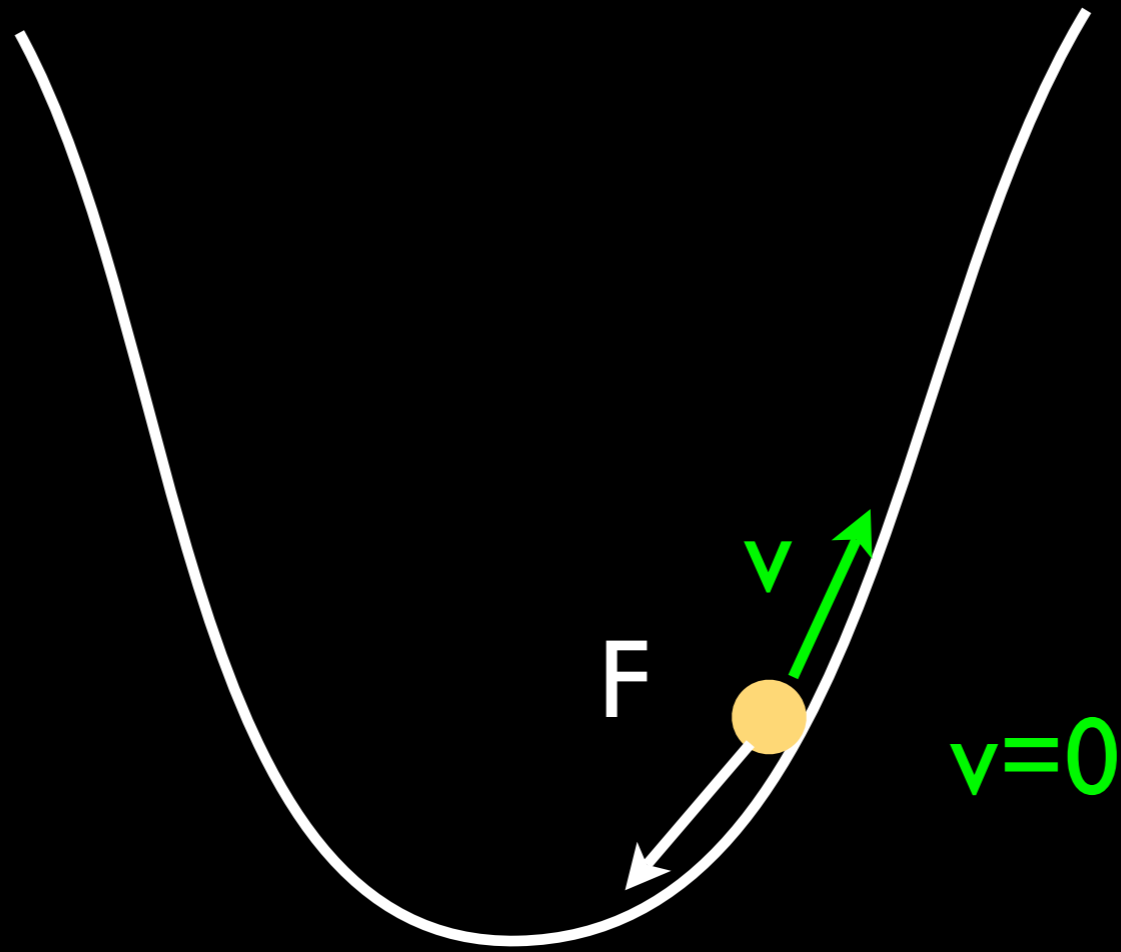
minimization: damped verlet

$$\frac{d^2 x_i(t)}{dt^2} = \frac{F_i}{m_i}$$

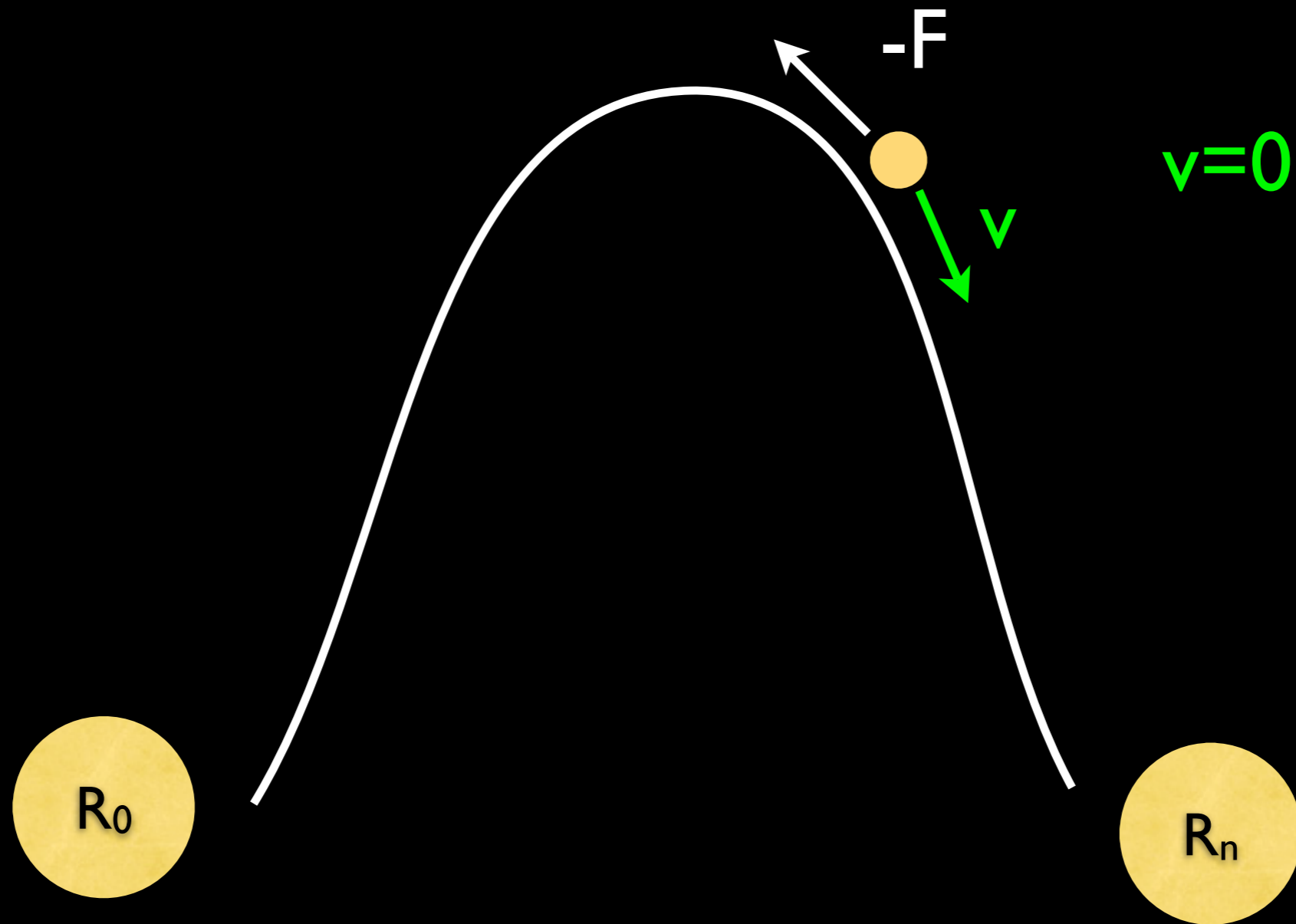
$$\frac{d^2 x_i(t)}{dt^2} \approx \frac{x_i(t + \Delta t) - x_i(t) - (x_i(t) - x_i(t - \Delta t))}{\Delta t^2}$$

$$x_i(t + \Delta t) - x_i(t) - (x_i(t) - x_i(t - \Delta t)) = \frac{F_i}{m_i} \Delta t^2$$

$$x_i(t + \Delta t) = 2x_i(t) - x_i(t - \Delta t) + \frac{F_i}{m_i} \Delta t^2$$

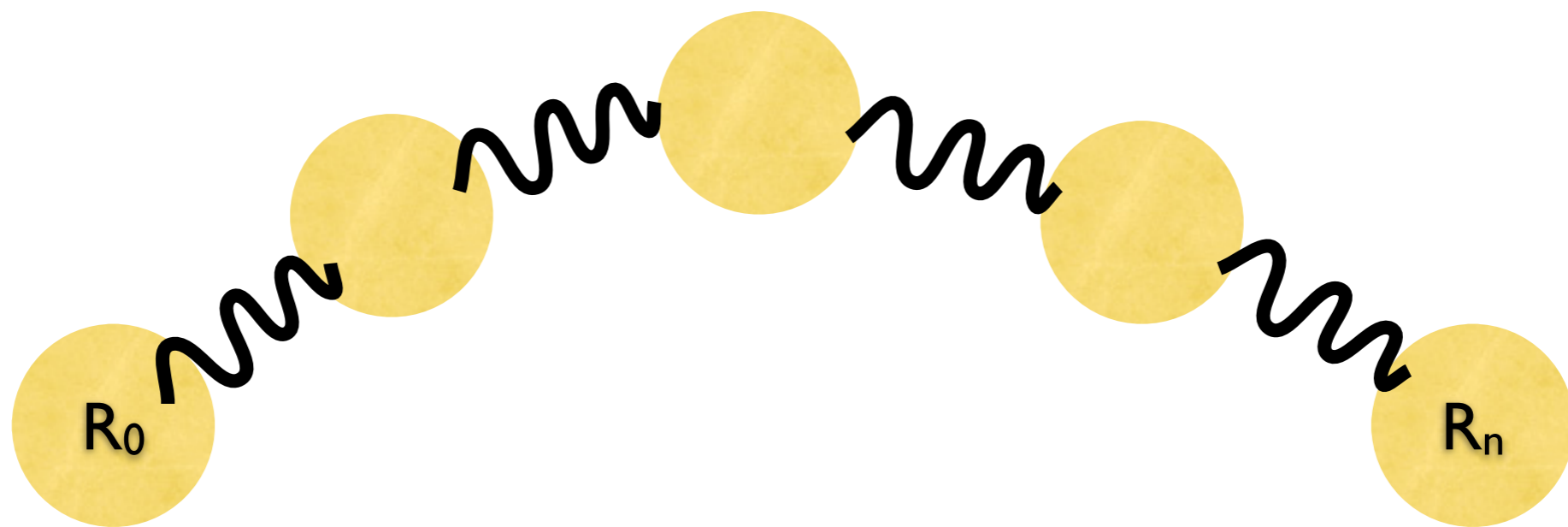


Drag method



NEB

$$R_i = R_0 + \frac{i}{n}(R_n - R_0)$$



$$S(R_1, R_2, R_3, \dots, R_n) = \sum_{i=1}^{n-1} E(R_i) + \sum_{i=1}^n \frac{k}{2} (R_i - R_{i-1})^2$$

NEB

$$F_i = -\nabla E(R_i) + k((R_{i+1} - R_i) - (R_i - R_{i-1}))$$

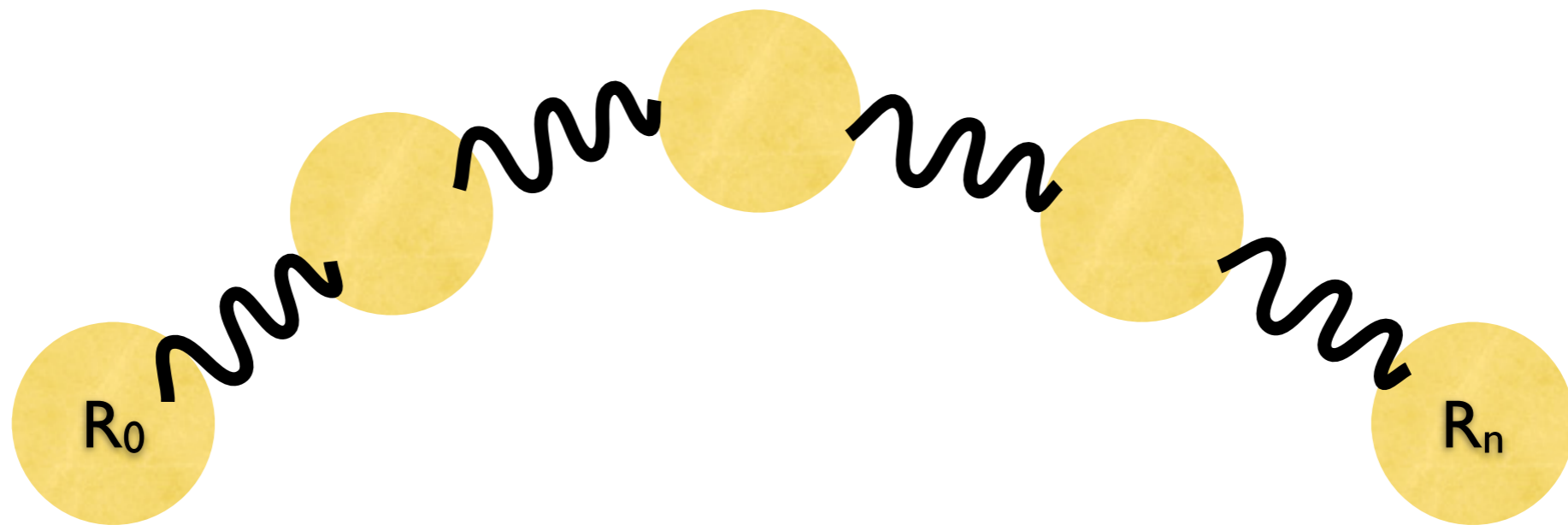
real force

Elastic force (F_s)

$$F'_i = -\nabla E(R_i)|_{perp} + F_s|_{paral} \rightarrow 0$$

INPUT PARAMETERS

$$R_i = R_0 + \frac{i}{n}(R_n - R_0)$$



$$S(R_1, R_2, R_3, \dots, R_n) = \sum_{i=1}^{n-1} E(R_i) + \sum_{i=1}^n \frac{k}{2} (R_i - R_{i-1})^2$$

?

- Como se escribe el vector tangente a la cadena en i ?

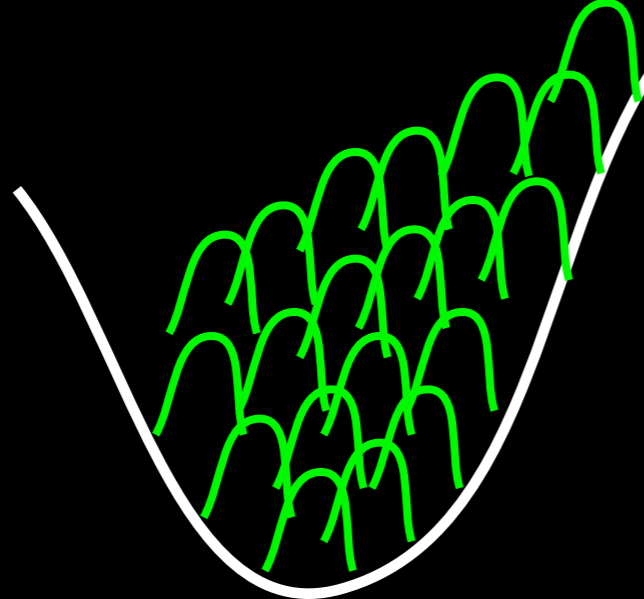
Climbing NEB

- “Highest” E configuration, spring force set to 0, real parallel force sign inverted.

Metadynamics

- “Constraint” dynamics.
- Explore the free energy as a function of a set of **Collective Variables** (CV).
- History-dependent random-walk (MD).
- A repulsive Gaussian is added each g^* MD steps.

“Fill the wells”



Collective variables s : distance between atoms, dihedral angle, ...

bias potential: “external potential”

$$V(\vec{s}, t) = \sum_{k\tau < t} W(k\tau) \exp \left(- \sum_{i=1}^d \frac{(s_i - s_i^{(0)}(k\tau))^2}{2\sigma_i^2} \right)$$

Collective variables s : distance between atoms, dihedral angle, ...

bias potential: “external potential”

$$V(\vec{s}, t) = \sum_{k\tau < t} W(k\tau) \exp \left(- \sum_{i=1}^d \frac{(s_i - s_i^{(0)}(k\tau))^2}{2\sigma_i^2} \right)$$

Height

Collective variables s : distance between atoms, dihedral angle, ...

bias potential: “external potential”

$$V(\vec{s}, t) = \sum_{k\tau < t} W(k\tau) \exp \left(- \sum_{i=1}^d \frac{(s_i - s_i^{(0)}(k\tau))^2}{2\sigma_i^2} \right)$$

width (standard deviation)

Collective variables s : distance between atoms, dihedral angle, ...

bias potential: “external potential”

position of the centroid

$$V(\vec{s}, t) = \sum_{k\tau < t} W(k\tau) \exp \left(- \sum_{i=1}^d \frac{(s_i - s_i^{(0)}(k\tau))^2}{2\sigma_i^2} \right)$$

Collective variables s : distance between atoms, dihedral angle, ...

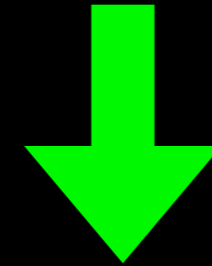
bias potential: “external potential”

$$V(\vec{s}, t) = \sum_{k\tau < t} W(k\tau) \exp \left(- \sum_{i=1}^d \frac{(s_i - s_i^{(0)}(k\tau))^2}{2\sigma_i^2} \right)$$

“history” dependence

Basic assumption:

$$\lim_{t \rightarrow \infty} V(s, t) \sim -F(s)$$



$$F(s) \sim \langle V(s, t) \rangle_t$$

time average

In a practical implementation the forces are modified

$$F = -\nabla V$$

Drag coordinate/CV examples

Moral de la historia

- Hay que saber muy bien que fenómeno se quiere estudiar y con que se quiere comparar.
- Las reacciones químicas/ la migration hay que elegir “inteligentemente” un camino inicial.
- El problema de N electrones que interaccionan es muy complejo. Hay que conocer muy bien el rango de validez de las aproximaciones.
- Sistema desordenado ... hay que cojerselo con filosofía y tanta tanta tila!!

email address and useful links

- lmartinsamos@gmail.com
- www.quantum-espresso.org
- www.yambo-code.org
- www.sissa.it